Summer Undergraduate Research Fellowships

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Fuzzy sets

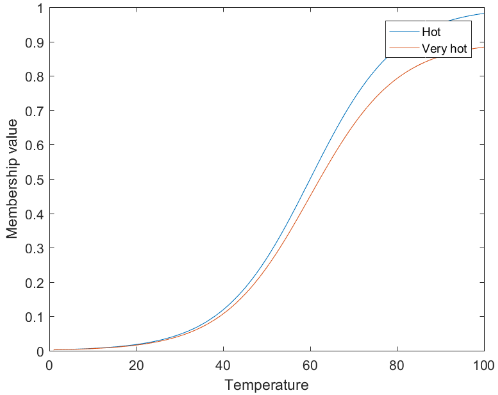
BASIC DEFINITIONS

Fuzzy set

$$100\,^\circ\mathrm{C} > 80\,^\circ\mathrm{C} > 50\,^\circ\mathrm{C}$$

$$\uparrow \qquad \uparrow \qquad \uparrow$$

$$>0.8 \qquad 0.8 \qquad < 0.8$$



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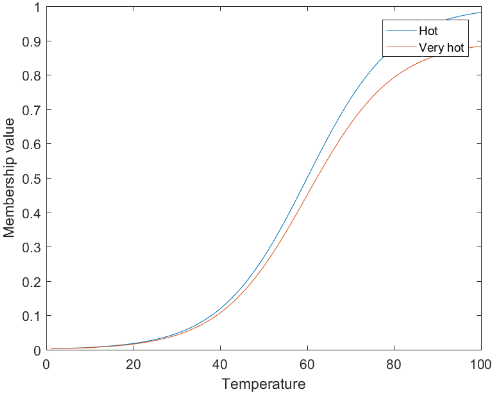

Operations on Fuzzy sets.

(1) (Union) $\mu_{A\cup B}(x) = max\{\mu_A(x), \mu_B(x)\}$

(2) (Intersection) $\mu_{A\cap B}(x) = min\{\mu_A(x), \mu_B(x)\}$

(3) (Complement) $\mu_{A^c}(x) = 1 - \mu_A(x)$

- For example, let
  $A = \{(x_1, 0.2), (x_2, 0.7), (x_3, 0.4), (x_4, 1)\}$,
  $B = \{(x_1, 0), (x_2, 0.9), (x_3, 0.5), (x_4, 0.3)\}$
- $A \cup B = \{(x_1, 0.2), (x_2, 0.9), (x_3, 0.5), (x_4, 1)\}$
- $A \cap B = \{(x_1, 0), (x_2, 0.7), (x_3, 0.4), (x_4, 0.3)\}$
- $A^c = \{(x_1, 0.8), (x_2, 0.3), (x_3, 0.6), (x_4, 0)\}$

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Definition.

- Let $\mathcal{F}$ be a $\sigma-$*algebra*. m is called a fuzzy measure on $(X, \mathcal{F})$ iff

(F1) $m(\phi) = 0$ when $\phi \in \mathcal{F}$

(F2) For $E \in \mathcal{F}, F \in \mathcal{F}$ and $E \subset F$, it implies $m(E) \leq m(F)$ (monotonicity)

(F3) For $\{E_n\} \subset F, E_1 \subset E_2 \subset \cdots$, and $\cup_{n=1}^{\infty} E_n \in \mathcal{F}$ imply $\lim_{n\to\infty} m(E_n) = m(\cup_{n=1}^{\infty} E_n)$ (continuity from below)

(F4) For $\{E_n\} \subset \mathcal{F}, E_1 \supset E_2 \supset \cdots, m(E_1) < \infty$ and $\cap_{n=1}^{\infty} E_n \in \mathcal{F}$ imply $\lim_{n\to\infty} m(E_n) = m(\cap_{n=1}^{\infty} E_n)$ (continuity from above).

No addition rule!

Definition.

- Let $\mathcal{F}$ be a $\sigma-algebra$ and $A \in \mathcal{F}$. The Sugeno integral of f on A with respect to m, which is denoted by $S\int_A f\,dm$, is defined by

$$S\int_A f\,dm \triangleq \sup_{\alpha\in[0,1]} [\alpha \wedge m(A \cap (f)_\alpha)],$$

where $(f)_\alpha = \{x|f \geq \alpha\}$ and $\vee$ and $\wedge$ denote maximum and minimum operators, respectively, i.e., $a \vee b = max(a, b)$ and $a \wedge b = min(a, b)$.

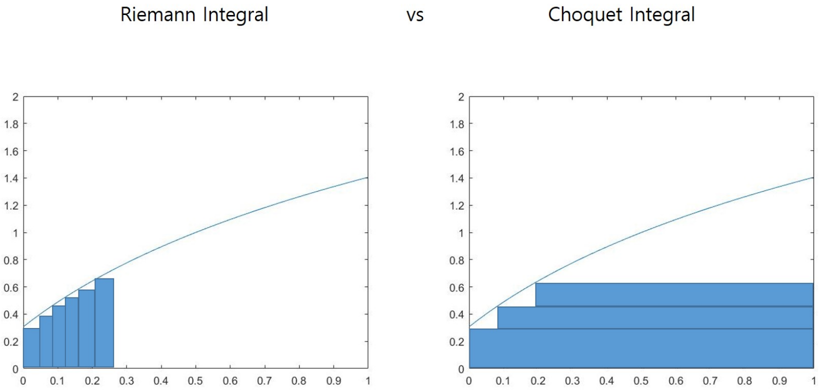
Definition.

- Let $\mathcal{F}$ be a $\sigma - algebra$ and $A \in \mathcal{F}$. The Choquet integral of f on A with respect to m, which is denoted by $C\int_A f dm$, is defined by

$$C\int_A f dm \triangleq \sum_{i=1}^{n}(f(x_i) - f(x_{i-1}))m(A_i),$$

where $\cdot_i$ states that the indices have been permuted so that $0 \leq f(x_1) \leq \cdots \leq (f)(x_n) \leq 1, f(x_0) = 0$ and $A_i \triangleq \{x_i, \cdots, x_n\}$.

Riemann Integral vs Choquet Integral



Similarity measure

Definition.

- A real function $s : \mathcal{F}^2 \to [0, \infty]$ is called a similarity measure, if s has following properties:

(S1) $s(A, B) = s(B, A), \quad \forall A, B \in \mathcal{F}$

(S2) $s(D, D^c) = 0, \quad \forall D \in \mathcal{P}(X)$

(S3) $s(C, C) = \max_{\forall A, B \in \mathcal{F}} s(A, B), \quad \forall C \in \mathcal{F}$

(S4) $A, B, C \in \mathcal{F}$, if $A \subset B \subset C$, then $s(A, B) \geqslant s(A, C)$ and $s(B, C) \geqslant s(A, C)$

Theorem.

- Let $A, B \in \mathcal{F}$ and m be a fuzzy measure on $X = \{x_1, x_2, \cdots, x_n\}$. Then

(I1) $S_S^d(A, B) = 1 - S\int f dm^d$,

(I2) $S_C^d(A, B) = 1 - C\int f dm^d$,

where $f(x) = |\mu_A(x) - \mu_B(x)|$, $m^d(E) = \frac{1}{n}\sum_{x\in E} x^{\frac{1}{d}}$ and $d \in \{1, 2, \cdots\}$, are similarity measure between A and B.

Theorem.

- Similarity measure based on distance measure. The nearer two sets are, the more similar they are.

(L1) $\sum_{i=1}^{n} |x_i - y_i|$

(L2) $\sqrt{\sum_{i=1}^{n} (x_i - y_i)^2}$

- Let $A = (1, 0, 0, 0)$, $B = (1, 1, 1, 1)$, and $C = (\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2})$, then
- $D_{L1}(A, C) = 2$
- $D_{L1}(B, C) = 2$

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- $D_{L1}(A, C) = 2$
- $D_{L1}(B, C) = 2$

Theorem.

- The first one is the correlation-based similarity measure. In this case, we calculate similarity between two items i and j, which is denoted by $w_{i,j}$.The correlation between item i and j will be
$$w_{i,j} = \frac{\sum_{u \in U}(r_{u,i} - \bar{r}_i)(r_{u,j} - \bar{r}_j)}{\sqrt{\sum_{u \in U}(r_{u,i} - \bar{r}_i)^2}\sqrt{\sum_{i \in I}(r_{u,j} - \bar{r}_j)^2}},$$
where $r_{u,i}$ is the rating of user u on item i, $\bar{r}_i$ is the average rating of the ith item by those users.

Theorem.

- The second one is vector cosine-based similarity. Let i and j be vectors of items which is purchased by users u and v. Then the vector cosine similarity between two users will be
$$w_{i,j} = cos(\overrightarrow{i}, \overrightarrow{j}) = \frac{\overrightarrow{i} \cdot \overrightarrow{j}}{\| \overrightarrow{i} \| \| \overrightarrow{j} \|},$$
where " $\cdot$ " denotes the dot-product of the two vectors.

Example

	i1	i2	i3	i4	i5	i6
A	5	5		4		
B		2	3		4	
C	4			4		2
D		3	3		3	

- $w_{A,C} = cos(\overrightarrow{a}, \overrightarrow{c}) = \frac{5\times4+4\times4}{\sqrt{5^2+5^2+4^2}\times\sqrt{4^2+4^2+2^2}} = 0.739$
- $w_{A,B} = cos(\overrightarrow{a}, \overrightarrow{b}) = \frac{5\times2}{\sqrt{5^2+5^2+4^2}\times\sqrt{2^2+3^2+4^2}} = 0.229$

Example

	i1	i2	i3	i4	i5	i6
A	1/3	1/3		-2/3		
B		-1	0		1	
C	2/3			2/3		-4/3
D	0	0	0		0	

- $w_{A,C} = cos(\overrightarrow{a}, \overrightarrow{c}) = \frac{\frac{1}{3} \times \frac{2}{3} - \frac{2}{3} \times \frac{2}{3}}{\sqrt{\frac{1}{3}^2 + \frac{1}{3}^2 + \frac{2}{3}^2} \times \sqrt{\frac{2}{3}^2 + \frac{1}{3}^2 + \frac{4}{3}^2}} = -0.179$
- $w_{A,B} = cos(\overrightarrow{a}, \overrightarrow{b}) = \frac{\frac{1}{3} \times -1}{\sqrt{\frac{1}{3}^2 + \frac{1}{3}^2 + \frac{2}{3}^2} \times \sqrt{1^2 + 0^2 + 1^2}} = -0.289$
- $w_{B,D} = cos(\overrightarrow{b}, \overrightarrow{d}) = 0.$

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